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Spin angular momentum of surface modes from the perspective of optical power flow

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We show that the spin angular momentum (SAM) carried by a surface mode can be linked to the expectation value, with respect to the distribution of optical power flow, of its decay constant by itself or divided by the product of permittivity and permeability of the medium. Rewriting the formulas for the SAM of a surface mode using the relation between the SAM density and the Poynting vector and then normalizing the light field so that the surface mode carries unit power, we derive novel formulas that show the linear relation between the SAM and those expectation values. The effect of propagation loss is also discussed briefly.

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Surface-mode lights propagating through interfaces between two media have captured interest of researchers, thanks to the advances in optical metamaterials [1,2]. Although they are usually thought to propagate only along the medium interfaces, the actual power flow through them is more complex: the optical power is transported in a vortex-like way [3]. This entails that surface modes carry angular momentum (AM).

Typically, the AM carried by an optical beam is longitudinal in the sense that it induces the rotation of light-absorbing particles around either the beam axis or their own axes which are parallel to it [4,5]. The AM of a surface-mode light is, however, a transverse one, i.e., its direction is normal to the propagating direction of the light. This unique feature has made it a topic of interest for the last few years [6–12]. Our group investigated the origin of this AM and showed that its spin component results from the rotation of the electric field comprising the surface mode [7,11]. We also proposed a geometrical interpretation that associates the total AM carried by a surface mode with the position of its power-flow center or the balance point of power flow through it [12]. (We note that a similar relation was first proposed in [13] for optical beams in free space.)

In this Letter, we approach the spin AM (SAM) of a surface mode from the perspective of optical power flow. Regarding the distribution of optical power flow as a kind of quasi-probability distribution and deriving expectation values with respect to it, we suggest a novel understanding that can link the SAM to the expectation value of either the decay constant or the decay constant divided by the product of permittivity and permeability of the medium. This understanding can provide a way of intuitive estimation of the qualitative characteristics of the SAM.

Let us consider a material interface shown in Fig. 1 with a normal vector $\hat{\mathbf{z}}$. We assume that it supports a TM-polarized, lossless surface mode along the $+x$ direction. The magnetic field of this mode can be written in a complex form as $\mathbf{H} = \hat{\mathbf{y}}\phi(z)\exp[j\omega\{(n_{\mathrm{eff}}/c)x - t\}]$, where ω and c are the angular frequency and speed of light in a vacuum, respectively, and $\phi(z)$ denotes the transverse profile of the mode whose effective index is n_{eff}. Putting the relative permittivity and relative permeability of the medium as ε_{ri} and μ_{ri} (the subscript i is 1 and 2 in the left- and right-side layers of the interface, respectively), $\phi(z)$ can be written as $\phi(z<0) = \exp(\kappa_1\omega z/c)$ and $\phi(z\geq 0) = \exp(-\kappa_2\omega z/c)$, where $\kappa_i = (n_{\mathrm{eff}}^2 - \varepsilon_{ri}\mu_{ri})^{1/2}$. The Poynting vector associated with this mode is given by [3]

$$\mathbf{p} = p_x\hat{\mathbf{x}} = \frac{n_{\mathrm{eff}}}{2c\varepsilon_0}\frac{\phi^2}{\varepsilon_r}\hat{\mathbf{x}}, \tag{1}$$

where ε_0 is the permittivity in a vacuum.

As is well known, there are two forms of linear momentum of light in a medium [14–19]. While the one, called the Abraham momentum, is inversely proportional to the refractive index of the medium, the other, the Minkowski counterpart, is

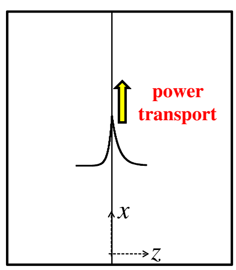

Fig. 1. Interface with a surface mode along the $+x$ direction.

linearly proportional to it. Despite the century-long efforts to determine whose version is correct and recent clarifications regarding some of their longstanding obscurities, the Abraham and Minkowski momenta are still among the most controversial and equivocal notions of optics. With the electromagnetic quantities, they are respectively given by $\mathbf{g}^A = \mathbf{e} \times \mathbf{h}/c^2$ and $\mathbf{g}^M = \mathbf{d} \times \mathbf{b}$, where $\mathbf{e}$ and $\mathbf{h}$ are the electric and magnetic fields, and $\mathbf{d}$ and $\mathbf{b}$ are the electric displacement and the magnetic flux density, respectively. Using the electric field $\mathbf{E}$ (in the complex form) of the above surface mode [7], we can derive its SAM densities based on Abraham and Minkowski momenta as

$$\mathbf{s}^A = \frac{\varepsilon_r \varepsilon_0}{2\omega} \operatorname{Im}\{\mathbf{E}^* \times \mathbf{E}\} = \frac{n_{\text{eff}}}{\omega c^2 \varepsilon_0} \frac{\kappa_z \phi^2}{\varepsilon_r^2 \mu_r} \hat{\mathbf{y}}, \tag{2}$$

$$\mathbf{s}^M = \frac{\varepsilon_0}{2\omega \mu_r} \operatorname{Im}\{\mathbf{E}^* \times \mathbf{E}\} = \frac{n_{\text{eff}}}{\omega c^2 \varepsilon_0} \frac{\kappa_z \phi^2}{\varepsilon_r} \hat{\mathbf{y}}, \tag{3}$$

where $\kappa_z = \kappa_1 - H(z)(\kappa_1 + \kappa_2)$. [$H(z)$ denotes the Heaviside step function that κ_z becomes positive and negative in $z < 0$ and $z \geq 0$, respectively].

Before moving to the next step, we would like to note that, in addition to the aforementioned Abraham-Minkowski controversy, there is another ambiguity regarding the definition of the SAM itself. The so-called electric-magnetic *duality* dictates that the SAM density should be proportional to $\frac{1}{2} \operatorname{Im}\{\mathbf{E}^* \times \mathbf{E} + \mathbf{H}^* \times \mathbf{H}\}$, rather than to $\operatorname{Im}\{\mathbf{E}^* \times \mathbf{E}\}$ [20–22]. In the case of a TM-polarized mode that we are now dealing with, we have $\mathbf{H}^* \times \mathbf{H} = 0$ since $\mathbf{H}$ has only a y-directional component. Therefore, the consideration of such duality simply adds a multiplication factor of 1/2 to Eqs. (2) and (3), entailing no significant change of physics investigated in this Letter.

By comparing Eqs. (2) and (3) with Eq. (1), we have

$$\mathbf{s}^{A(M)} = \frac{\lambda}{\pi} \frac{\kappa_z}{\gamma^{A(M)}} \frac{p_x}{c^2} \hat{\mathbf{y}}, \tag{4}$$

where λ is the wavelength of light in a vacuum, and γ becomes either $\gamma^A = \varepsilon_r \mu_r$ or $\gamma^M = 1$. The SAM per unit length along the propagation direction (hereafter simply SAM) is defined by $\mathbf{S}^{A(M)} = \int \mathbf{s}^{A(M)} \mathrm{d}z$. Normalizing $\mathbf{S}^{A(M)}$, as in the case of the total AM [12], we can obtain the Abraham and Minkowski SAMs of the surface mode *per unit power* as

$$c^2 \mathbf{S}_0^{A(M)} = \hat{\mathbf{y}} \frac{\lambda}{\pi} \frac{\int \frac{\kappa_z}{\gamma^{A(M)}} p_x \mathrm{d}z}{\int p_x \mathrm{d}z} = \frac{\lambda}{\pi} \left\langle \frac{\kappa_z}{\gamma^{A(M)}} \right\rangle \hat{\mathbf{y}}, \tag{5}$$

where $\langle f(z) \rangle$ denotes the expectation value of f with respect to the distribution of optical power flow (we thus regard its distribution as a kind of quasi-probability distribution). Equation (5) gives us a simple interpretation of the Abraham and Minkowski SAMs carried by a surface mode: they can be linked to the expectation value of $\kappa_z/(\varepsilon_r \mu_r)$ or κ_z from the viewpoint of optical power flow.

Equation (5) can be further simplified by the following relation which holds when $f(z < 0) = f_1$ and $f(z \geq 0) = f_2$:

$$\langle f \rangle = \frac{f_1 - \zeta_{21} f_2}{1 - \zeta_{21}}, \tag{6}$$

where ζ_{21} denotes the ratio of the amount of power flow in the right-side layer to that in the left-side layer (i.e., $|\int_0^\infty p_x \mathrm{d}z| / |\int_{-\infty}^0 p_x \mathrm{d}z|$) and can be written as $\zeta_{21} = \varepsilon_{r1}^2/\varepsilon_{r2}^2$. We can see that this power-ratio parameter also plays a critical role in the evaluation of the expectation value and thus in the determination of the SAMs. It is notable that Eq. (6) results in (refer to [23] for the definition of n_{eff})

$$\langle \varepsilon_r \mu_r \rangle = \frac{\varepsilon_{r1}\mu_{r1} - \zeta_{21}\varepsilon_{r2}\mu_{r2}}{1 - \zeta_{21}} = n_{\text{eff}}^2, \tag{7}$$

which provides a new meaning for n_{eff}: n_{eff}^2 is none other than the expectation value of $\varepsilon_r \mu_r$ with respect to the distribution of optical power flow.

To check the validity of this discussion, we calculated actual SAMs of surface modes propagating through three different types of interfaces: between doubly negative and doubly positive, between ε-negative and doubly positive, and between ε-negative and μ-negative materials (DNG-DPG, ENG-DPG, and ENG-MNG cases, respectively). Their detailed parameters are given in the caption of Fig. 2. Their left/right layers have negative/positive ε_r, with the major forward power flowing in the right-side layers [i.e., $\zeta_{21} > 1$; see Fig. 3(a)]. The effective indexes of surface modes along these interfaces are nearly the same (~2.2), while the power ratio ζ_{21} of the ENG-DPG case is quite larger than those of the other cases.

Calculation results are shown in Fig. 2. We may anticipate several qualitative aspects of the Minkowski SAM from $\langle \kappa_z \rangle$. (1) Since the major power flow takes place in the right-side layer, $\langle \kappa_z \rangle$ and thus S_0^M ($= \mathbf{S}_0^M \cdot \hat{\mathbf{y}}$) will be negative in all three cases (please note that κ_z is negative in $z \geq 0$). (2) Given the same effective index, $|\kappa_z|$ is larger in singly negative media than in DNG or DPG materials. Therefore, $|S_0^M|$ of the ENG-MNG case will be enhanced compared with that of the DNG-DPG case. (3) $|S_0^M|$ of the ENG-DPG case, when compared with that of the DNG-DPG case, will be diminished because of the increase in ζ_{21} [see Eq. (6) and note that $\zeta_{21} > 1$]. Although $|\kappa_z|$ in the left-side layer is increased as well, its effect is much smaller than that of ζ_{21}. All these characteristics can be identified in Fig. 2, which clearly demonstrates the validity of our discussion above.

Next, for the comparison of the Abraham SAM with the Minkowski counterpart, let us introduce $\xi_i = \kappa_i/(\varepsilon_{ri}\mu_{ri})$ ($i = 1$ and 2 in the left- and right-side layers) and $\xi_z = \xi_1 - H(z)(\xi_1 + \xi_2)$. Please remember that κ_1 and κ_2

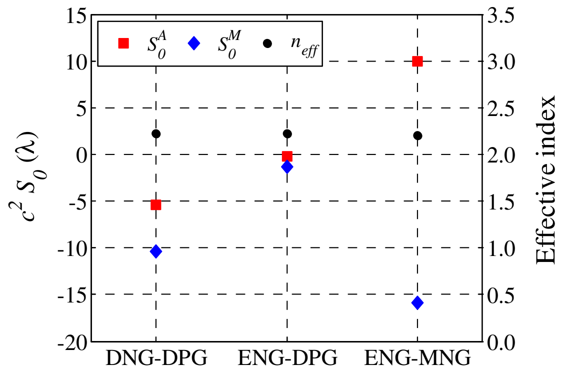

Fig. 2. Abraham and Minkowski SAMs (as well as effective indexes) of three surface modes which are formed near the interfaces between DNG ($\varepsilon_r = -2.2$ and $\mu_r = -0.8$) and DPG (silica), between ENG ($\varepsilon_r = -3.6$ and $\mu_r = 1.0$) and DPG (silica), and between ENG ($\varepsilon_r = -2.0$ and $\mu_r = 1.0$) and MNG ($\varepsilon_r = 1.9$ and $\mu_r = -0.7$) media. We assumed $\lambda = 1550$ nm.

are both positive, and κ_z becomes positive and negative in $z < 0$ and $z \geq 0$, respectively. By comparing $\langle\xi_z\rangle$ with $\langle\kappa_z\rangle$, we can also anticipate the following results: (1) In the DNG-DPG case, the sign of each ξ_i remains the same as that of κ_i, only its magnitude being a half of $|\kappa_i|$ (since $\varepsilon_{r1}\mu_{r1} \sim \varepsilon_{r2}\mu_{r2} \sim 2$). Therefore, S_0^A will be roughly $S_0^M/2$. (2) In the ENG-MNG case, the sign of each ξ_i is reversed (and thus becomes negative) with its magnitude smaller than $|\kappa_i|$ by a factor of $|\varepsilon_{r1}\mu_{r1}| = 2$ or $|\varepsilon_{r2}\mu_{r2}| = 1.33$. We can thus expect that $S_0^A \cdot S_0^M < 0$ and $|S_0^A|$ will be reduced but not as much as a half of $|S_0^M|$. (3) In the ENG-DPG case, only the sign of ξ_1 is reversed while $|\xi_1|$ and $|\xi_2|$ become smaller than $|\kappa_1|$ and $|\kappa_2|$, respectively. The effect of ξ_1 is rather complex to estimate. However, since the portion of the major power flow in the right-side layer is significantly large, as ζ_{21} represents in this case, its effect is not dominant. Therefore, we can roughly take into account only the effect of ξ_2 and presume that S_0^A will be still negative and $|S_0^A| < |S_0^M|$. We can confirm these features in Fig. 2 as well. These results and discussion prove that our understanding of the SAM as a quantity related to $\langle\kappa_z\rangle$ (or $\langle\xi_z\rangle$) is quite effective in the extraction of its qualitative characteristics.

Let us now apply Eq. (5) further to the *backward* surface modes whose effective indexes are negative. As in the cases of Fig. 2, we will consider as examples the interfaces whose left/right layers have negative/positive ε_r. From the perspective of optical power flow, what changes when the mode becomes backward is where the major power flow takes place: we have $\zeta_{21} < 1$ and the major forward power comes to flow in the left side of the interface [see Fig. 3(b)]. Therefore, we can presume that $\langle\kappa_z\rangle$ and thus S_0^M become positive (please remember κ_z is positive in $z < 0$). The sign of S_0^A remains the same as that of S_0^M if $\varepsilon_{r1}\mu_{r1}$ is positive, but is reversed when $\varepsilon_{r1}\mu_{r1}$ is negative. These characteristics can be recognized in Fig. 3(c) where we

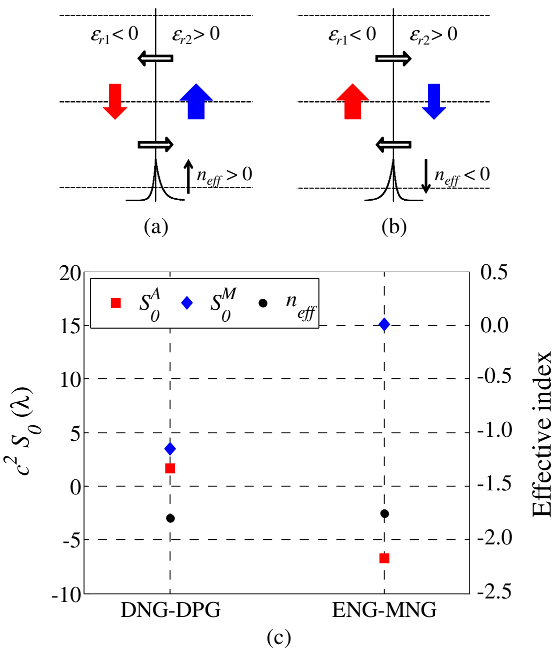

Fig. 3. (a)–(b) Schematics showing instantaneous power flow via forward and backward surface modes, respectively. (c) The same as Fig. 2 for backward modes at the interfaces between DNG ($\varepsilon_r = -1.9$ and $\mu_r = -1.2$) and DPG (silica), and between ENG ($\varepsilon_r = -2.0$ and $\mu_r = 1.0$) and MNG ($\varepsilon_r = 2.1$ and $\mu_r = -1.2$) media.

showed the calculated SAMs of two backward modes at DNG-DPG and ENG-MNG interfaces (detailed parameters can be found in the caption). This also shows that our understanding proposed in this Letter is useful in the estimation of the qualitative features of the SAM carried by surface modes.

Up to this point, we have neglected the losses of materials. If we take them into account, the magnetic field of the surface mode can be written as $\mathbf{H} = \hat{\mathbf{y}}\phi(z)\exp[j\omega\{(n_{\text{eff}}/c)x - t\}]$ $\exp(-\alpha\omega x/c)$, where we have written the complex propagation constant as $\beta = (n_{\text{eff}} + j\alpha)\omega/c$ (α is always nonnegative). The complex mode profile $\phi(z)$ is given by $\phi(z < 0) = \exp(\kappa_1^r\omega z/c)\exp(j\kappa_1^j\omega z/c)$ and $\phi(z \geq 0) = \exp(-\kappa_2^r\omega z/c)$ $\exp(-j\kappa_2^j\omega z/c)$, where we assumed $\kappa_i = \kappa_i^r + j\kappa_i^j$. [Hereafter, the superscripts r and j denote the real and imaginary parts of the corresponding quantity, respectively.] Then, we have [3]

$$p_x = \frac{e^{-z/L_x}}{2c\varepsilon_0}\frac{(n_{\text{eff}}\varepsilon_r^r + \alpha\varepsilon_r^j)|\phi|^2}{|\varepsilon_r|^2}, \quad (8)$$

where $L_x = (2\alpha\omega/c)^{-1}$. The Abraham and Minkowski SAM densities are given by [7]

$$\mathbf{s}^{A(M)} = \frac{e^{-z/L_x}}{\omega c^2\varepsilon_0}\frac{\varepsilon_r^r(n_{\text{eff}}\kappa_z^r + \alpha\kappa_z^j)|\phi|^2}{\gamma_l^{A(M)}|\varepsilon_r|^2}\hat{\mathbf{y}}, \quad (9)$$

where γ_l becomes either $\gamma_l^A = \varepsilon_r^r|\mu_r|^2/\mu_r^r$ or $\gamma_l^M = 1$. Since we are now limiting our attention to the cases where the losses are not so severe and, thus, $|(\alpha/n_{\text{eff}})(\varepsilon_r^j/\varepsilon_r^r)| \ll 1$, Eq. (9) leads us to

$$c^2\mathbf{S}_0^{A(M)} = \frac{\lambda}{\pi}\left[\left\langle\frac{\kappa_z^r}{\gamma_l^{A(M)}}\right\rangle + \frac{\alpha}{n_{\text{eff}}}\left\langle\frac{\kappa_z^j}{\gamma_l^{A(M)}}\right\rangle\right]\hat{\mathbf{y}}. \quad (10)$$

Equation (10) demonstrates that our discussion in the lossless case can be extended easily to the lossy case: the SAMs come to be proportional not only to $\langle\kappa_z^r/\gamma_l\rangle$ but also to $\langle\kappa_z^j/\gamma_l\rangle$ with a reduction coefficient of α/n_{eff}.

Finally, we would like to comment on the experiment for the verification of the discussed properties of the SAM carried by a surface mode. As was discussed already in several literatures, the SAM can manifest itself in the mechanical action on probe objects [6,22] or microparticles [8,10]. In these cases, however, its measurement is somewhat complicated because of the additional mechanical action via the orbital AM of the surface mode. Another way is to adopt a Stern-Gerlach-like configuration. As was demonstrated in [24], when an incident light excites polaritons in a medium, its SAM can induce nonzero magnetic moments of those polaritons. They can thus interact with a magnetic field gradient and undergo deflection. Since a surface-mode light is coupled to polaritons in most cases, its nonvanishing SAM can entail nonzero magnetic moments of coupled polaritons, and the exertion of a magnetic field gradient can deflect the surface mode. By measuring this deflection, we can get the actual SAM of the surface mode independently of its orbital AM.

The importance of our novel understanding can be appreciated well from the *designing* point of view. For instance, if we want to implement a material interface that supports a surface mode whose Minkowski SAM is, say, along the $+y$ direction for some reason, what should we do? Since we now have $\mathbf{S}_0^M \propto \langle\kappa_z\rangle\hat{\mathbf{y}}$, such a SAM can be realized easily by having $\zeta_{21} < 1$ or making the major power of the surface mode flow in the left-side layer of the interface. Can we go further so that its

Abraham counterpart is along the $-y$ direction? Yes, with the use of singly negative media as the left-side layer since $\mathbf{S}_0^A \propto \langle \kappa_z/\varepsilon_r\mu_r\rangle\hat{\mathbf{y}}$. Another example is related to the total AM of the surface mode. Can we devise a surface mode whose (Abraham) SAM is parallel with its total AM ($\mathbf{J}_0^A$)? (We have $c^2\mathbf{J}_0^A = \hat{\mathbf{y}}\langle z - z_0\rangle$, where z_0 indicates the coordinate of the reference point with respect to which the total AM is defined [12].) With $z_0 = 0$, the answer is yes: if $\varepsilon_r\mu_r$ of the layer where the major power flows is negative, we can obtain $\mathbf{J}_0^A \cdot \mathbf{S}_0^A \propto \langle z\rangle\langle \kappa_z/\varepsilon_r\mu_r\rangle > 0$ since $\langle z\rangle\langle \kappa_z\rangle < 0$ [25]. These illustrations show that our understanding can provide an essential guideline for the design of surface modes with desired SAM characteristics. Actually, required characteristics will depend on the specific applications. We hope future studies can draw such desired properties of the SAM and our findings will be of use for their realization.

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